values found for the q^*_j into Eqs. (16) and (17) gives an approximate solution of problem (3)-(12).

Figure 2 shows the results of a numerical calculation of the temperature distribution in the system calculated on a BÉSM-6 computer for the following values of the parameters: $\lambda_1 = 200$ W/m•deg, $\lambda_2 = 100$ W/m•deg, $a = 2 \cdot 10^{-3}$ m, $b = 4 \cdot 10^{-3}$ m, $c = 10^{-2}$ m, $d = 2 \cdot 10^{-2}$ m, $\alpha = 10^4$ W/m²•deg, $f_0 = 10^5$ W/m², M = 5. The lower limit of the variation of the variables was taken as $q^L_j = -f_0$, the upper limit $q^U_j = f_0$, and the initial approximation $q_j = 0$.

In order to compare the approximate solution found by the scheme described by steps 1-4 with the analytical solution, problem (3)-(12) was solved for d = c. The maximum discrepancy between the analytical solution and that obtained by using the proposed scheme does not exceed 3.3% (Table 1). The solution of this problem by the net-point method using the complex of codes of the KSI-BÉSM-6 program [5] is much inferior to the proposed method in the expenditure of machine time for the same accuracy of the solution.

NOTATION

U_i, U_j, temperature distribution functions in i-th and j-th regions; n, unit vector in direction of outward normal to boundary of region; λ_i, λ_j, thermal conductivities of i-th and j-th regions; L, number of matching boundaries; x, y, axes of Cartesian coordinate system; α, heat-transfer coefficient; f(x), heat flux distribution function; M, number of coefficients in step function representation of q(x).

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APPLICATION OF DIMENSIONAL ANALYSIS TO THE PROBLEM OF THE ACTION OF ULTRASOUND ON AIR IN A CAPILLARY TUBE

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By using dimensional analysis an expression is obtained for the air pressure change in a capillary tube channel caused by the effect of ultrasound. A comparison with experimental results is performed.

An analysis of the dimensionality of the quantities governing a physical phenomenon affords a possibility, in a number of cases, of obtaining characteristic relationships comparatively easily, which connect these quantities in a mathematical description of the phenomenon [1]. Dependences of the air pressure in channels of dead-end capillary tubes placed upright at a short distance from the concentrator on different parameters were experimentally obtained in [2]. It was shown that the main variables on which the P in the channel depends are the amplitude of the concentrator A displacement and the frequency of the ultrasonic oscillations f, the inner tube diameter d, and its wall thickness Δ, as well as the magnitude of the effective gap δ^*, which equals the distance between the tube endface and the lower position of the oscillating concentrator. Moreover, characteristics of the medium in which the ultrasonic oscillations are propagated and the flow which causes the change in pressure

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in the capillary tube channel should be included. In this case this is the air density ρ and viscosity μ, as well as the velocity of sound propagation c therein.

It is seen from the above that P depends on eight variable parameters. If the mass M, time T, and length L are taken as fundamental units of measurement in analyzing the dimensionality of the phenomenon under examination, then to express P by using them will evidently depend on five dimensionless criteria. So great a quantity of criteria makes this expression hardly suitable for practical application. Hence, the increase in the number of fundamental measurement units is quite urgent.

Two kinds of processes occur in our experiments: acoustic, i.e., the propagation of ultrasonic oscillations, and aerodynamic, i.e., the flow of this medium in a tube channel and in the gap between the concentrator and the tube. Hence, we can formally introduce two length and time measurement scales, which refer to the acoustic and aerodynamic processes. Let T_a and T denote the time measurement scales in acoustics and aerodynamics, respectively. Let us analogously introduce the length measurement scales L_a and L. Moreover, we use the extension of dimensional analysis [3] associated with the introduction of several mutually independent length measurement units in different directions. In the problem under consideration, the axial (z) and radial (r) directions in a cylindrical coordinate system are such directions. Hence, the mass M, times T and T_a, and also lengths L_z, L_r, and L_a are the fundamental measurement units in our problem. Let us note that in this case there is no separation of the length measurement scales into directions in the acoustic process since ultrasonic oscillations act mainly in the axial direction.

To compose the dimensionality equation, let us use the energy intensity of the ultrasonic oscillations I, which has a deeper physical meaning, in place of the displacement amplitude A. Let us write the dimensions of the quantities in the problem:

$$[P] = ML_zL_r^{-2}T^{-2};\ [I] = ML_aL_zL_r^{-2}T_a^{-2}T^{-1};\ [f] = T_a^{-1};\ [\rho] = ML_z^{-1}L_r^{-2};$$
$$[c] = L_aT_a^{-1};\ [d] = L_r;\ [\mu] = MT^{-1}L_z^{1-\alpha}L_r^{\alpha-2};\ [\delta^*] = L_z;\ [\Delta] = L_r.$$

Let us clarify the writing, e.g., of the dimensionality of the dynamic viscosity coefficient μ. For the axial and radial flows taking place in the experiments, the dimensionality of μ is written as $[\mu]_z = MT^{-1}L_z^{-1}$ and $[\mu]_r = MT^{-1}L_zL_r^{-2}$. The relationship between the axial and radial flows changes as each of the quantities δ*, Δ, or d varies; hence, a variable exponent characterizing this relationship should enter into the dimensionality of μ. It can be assumed that in the general case

$$[\mu] = \{([\mu]_r)^m\,([\mu]_z)^n\}^{\frac{1}{m+n}},$$

where n and m are any real numbers (n ≠ m). Hence,

$$[\mu] = (ML_zL_r^{-2}T^{-1})^{\frac{m}{n+m}}(ML_z^{-1}T^{-1})^{\frac{n}{n+m}} = MT^{-1}L_z^{\frac{m-n}{m+n}}L_r^{-\frac{2m}{m+n}} = MT^{-1}L_z^{1-\alpha}L_r^{\alpha-2},$$

where α is any real number.

The pressure P can be represented in the form

$$P = I^af^b\rho^kc^md^n\mu^q\,\varphi(M,\ N),$$

where φ(M, N) is a function dependent on two dimensionless criteria M and N which are not yet known. Now, let us form the dimensionality equation

$$ML_zL_r^{-2}T^{-2} = (ML_aL_zL_r^{-2}T_a^{-2}T^{-1})^aT_a^{-b}(ML_z^{-1}L_r^{-2})^kL_r^n\,(MT^{-1}L_z^{1-\alpha}L_r^{\alpha-2})^q\,(L_aT_a^{-1})^m.$$

After standard procedures used in dimensional analysis, we obtain

$$P = \left(\frac{I^{2\alpha-2}f^{2-2\alpha}\mu^2}{\rho^\alpha d^{2\alpha}c^{2\alpha-2}}\right)^{\frac{1}{\alpha}}\varphi\left(\frac{\Delta}{d},\ \frac{Id^\alpha}{fc\mu\delta^{*\alpha}}\right), \qquad (1)$$

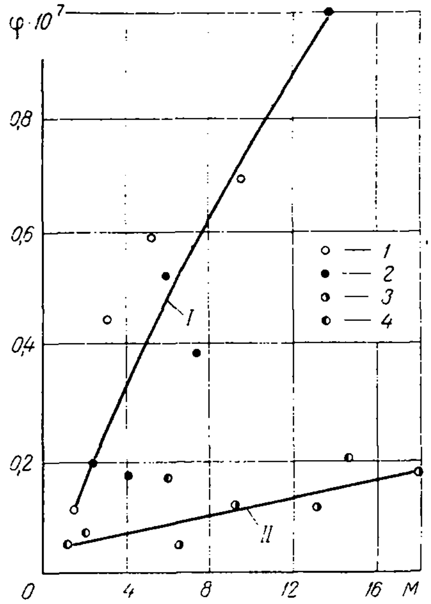

Fig. 1. Dependence of the function φ on M for N = 5 [curve I: 1) 21.7 and 2) 41.9 kHz] and N = 7.25 [curve II: 3) 21.7 and 4) 41.9 kHz].

which admits of experimental confirmation. Independently of the value of the unknown exponent α, the pressure becomes equal to P/n^2 for a simultaneous n-times change in the quantities d, Δ, and δ*. A series of experiments performed with capillary tubes with d = 0.2-1.2 mm and Δ = 0.5–6.5 mm showed good agreement between this deduction and the experimental results.

Let us now make the following assumption. Let us assume that the energy intensity of the ultrasonic oscillations is expressed, in our case, by the formula obtained for a plane wave

$$I = 2\pi^2 A^2 f^2 \rho c.$$

Then (1) is converted into an expression in which only the quantities being measured and those tabulated enter:

$$P = \left(\frac{A^{4\alpha-4} f^{2\alpha-2} \mu^2}{\rho^{\alpha-2} d^{2\alpha}}\right)^{\frac{1}{\alpha}} \varphi\left(\frac{\Delta}{d}, \frac{A^2 f \rho d^{\alpha}}{\mu \delta^{*\alpha}}\right). \tag{2}$$

Therefore, the similarity criteria of the problem under consideration are the following dimensionless variable complexes:

$$N = \Delta/d \text{ and } M = A^2 f \rho d^{\alpha}/\mu\delta^{*\alpha}.$$

Let us note that α cannot be zero since otherwise the pressure dependence on the gap δ* existing in experiments would drop out and also the variability in the sign of the function φ, resulting from the experimental fact of the existing of both an excess pressure and rarefaction depending on the quantities d and Δ observed in [2], vanishes. The case α = 2 corresponds to taking account of only the axial air flow. It can be assumed that the flow in this direction acquires greater and greater value as the gap increases; hence, we have constructed a dependence of the function φ on the criterion M for different values of N for this case. Results of experiments performed at the frequencies 21.7 and 41.9 kHz for capillary tubes with d = 0.2-1.2 mm, Δ = 0.5-6.5 mm and for δ* > 5A were used. The curves presented in the figure indicate the satisfactory agreement between the expression obtained and experiment.

An analysis of the experimental results obtained with thick-walled tubes for δ* < 5A, when the air flow direction in the gap varies for definite values of N and changes in A or δ*, indicates the presence of an extremum (at least one) in the function φ depending on M. Thus, e.g., the dependence of the pressure on δ* has a maximum in the case N > 5 and Δ > 5, the other parameters being constant. However, the question of the specific value of α suitable for practical utilization for δ* < 5A remains open.

NOTATION

P, pressure; A, amplitude of the concentrator displacement; f, frequency; I, energy intensity of the ultrasonic oscillations; d, inner diameter; Δ, wall thickness of the capillary tube; δ*, magnitude of the effective gap; ρ, density; μ, coefficient of dynamic viscosity; c, speed of sound in air.

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SPEED OF ULTRASOUND IN WATER OVER A WIDE RANGE OF PRESSURE AND TEMPERATURE

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UDC 534.22

Experimental data are used to derive a formula for determination of the speed of sound in water over a wide range of state parameters.

The speed of sound in water was studied over a wide range of temperature and pressure in [1], which presented its experimental results in the form of a table for the pressure range of 3-30°C at pressures to 70 MPa, and from 75 to 374°C for pressures to 50 MPa. As the authors noted, the method proposed therein allows determination of the speed of sound in water with quite high accuracy.

The present author has attempted to use the experimental data of [1] to define the speed of sound in water as an analytical function of temperature and density.

The speed of sound in liquid n-alkanes [2] has been described by a formula

$$u = u_s' + B(\rho - \rho_s). \quad (1)$$

Tests showed that the speed of sound isotherms in water as a function of $(\rho - \rho_s)$, according to Eq. (1) for 11 isotherms (0, 10, 20, 30, 100, 130, 150, 200, 250, 300, and 350°C) presented in the study, were straight lines. The ρ values for these isotherms at corresponding pressures, presented in Table 1, were calculated from the equation of the isotherm [3]

$$\frac{pv}{RT} = 1 + B\rho + E\rho.^4 \quad (2)^*$$

It should be noted that the specific volumes calculated with Eq. (2), as is evident from Table 2, agree quite well with the tolerances of the International Table for water and water vapor [4].

The saturated water densities were taken from [4], since Eq. (2) does not provide the required accuracy for ρ_s at high temperatures.

Commencing from the linearity of the isotherms, according to Eq. (1), the least-squares method was used to find the speed of sound values u_s' for saturated water and the acoustical coefficient B for all 11 isotherms.

Considering the complex form of the curves $u_s' = f(t)$ and $B = \varphi(t)$ shown in Fig. 1, it was necessary to employ polynomials for their description with satisfactory accuracy. These polynomials are easily solved by a Horner type $\Sigma a_i t^i$ system. Seventh-order polynomials were

*For water, R = 4.6151 bar•cm³/g/deg.

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